**Efficient Sudoku Generation using Backtracking**

**A PROJECT REPORT**

**Submitted by**

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*in partial fulfilment for the completion of course*

**CSA0697-Design and Analysis of Algorithms for Amortized Analysis**



**SIMATS ENGINEERING**

**THANDALAM**

**SEPTEMBER 2024**

**BONAFIDE CERTIFICATE**

Certified that this project report titled “**Efficient Sudoku Generation using Backtracking**” is the bonafide work V.POOJITHA (192210132) , who carried out the project work under my supervision as a batch. Certified further, that to the best of my knowledge, the work reported here in does not form any other project report.

Project Supervisor Head of the Department

Date: Date:

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**ABSTRACT**

Sudoku puzzles are popular logic-based games that require filling a 9x9 grid with digits from 1 to 9 such that each row, column, and 3x3 sub grid contains all numbers exactly once. This paper explores an efficient approach to generate Sudoku puzzles using the backtracking algorithm implemented in Python. The algorithm systematically fills the Sudoku grid while adhering to Sudoku rules, leveraging recursive backtracking to explore possible solutions and backtrack when a conflict arises. Key aspects include initializing the grid, filling the diagonal sub grids, and recursively attempting to place digits in empty cells until a complete puzzle is generated. The presented method not only ensures the generation of valid Sudoku configurations but also provides insights into the application of backtracking in solving constraint satisfaction problems.

**Keywords:** Sudoku, Backtracking algorithm, Constraint satisfaction, Python programming, Puzzle generation, Logic games, Algorithmic approach, Recursive algorithms.

**PROBLEM STATEMENT AND ASSUMPTIONS:**

The goal is to develop an algorithm to generate a valid Sudoku puzzle using the backtracking technique. The algorithm should create a complete and valid Sudoku board and then remove a specified number of cells to create a playable puzzle while ensuring that there is a unique solution to the generated puzzle.

 **Sudoku Board Generation**:

* The Sudoku board must be a 9x9 grid.
* Each number from 1 to 9 must appear exactly once in each row, each column, and each of the nine 3x3 sub-grids.

 **Puzzle Creation**:

* The algorithm should generate a complete Sudoku board first, ensuring it is valid.
* After generating a complete board, it should randomly remove a specified number of digits (K) to create the Sudoku puzzle.
* The resulting puzzle should have a unique solution.

 **Backtracking Algorithm**:

* The backtracking algorithm must be used to fill the Sudoku board and to ensure all constraints of Sudoku are met.
* If a conflict arises during the placement of numbers, the algorithm should backtrack and try different possibilities.

**Assumptions**

1. **Grid Size**:
   * The algorithm assumes a standard 9x9 Sudoku grid.
2. **Uniqueness**:
   * The generated puzzle must have a unique solution. This is a key characteristic of valid Sudoku puzzles.
3. **Number of Digits to Remove (K)**:
   * The number K of digits to remove should be a parameter of the algorithm. It should be less than 81 (the total number of cells in the grid) and should be chosen such that the puzzle remains solvable.
4. **Randomness**:
   * The algorithm may involve randomization in selecting which numbers to place or which cells to remove, ensuring that the generated puzzles are varied.
5. **Performance**:
   * The algorithm should be efficient enough to generate a complete Sudoku puzzle and create a playable puzzle in a reasonable amount of time.
6. **Input/Output**:
   * The program will take as input the value of K (the number of cells to remove) and will output the generated Sudoku puzzle along with the complete board for verification.

**INTRODUCTION**

Sudoku puzzles, with their simple rules yet complex solving strategies, have captivated puzzle enthusiasts worldwide. Generating Sudoku puzzles programmatically involves ensuring each grid adheres to strict rules: every row, column, and 3x3 sub grid must contain digits from 1 to 9 exactly once. Among various methods, the backtracking algorithm stands out for its efficiency in generating valid Sudoku configurations. This algorithm explores potential solutions recursively, backtracking when a conflict arises, ensuring each puzzle is not only solvable but also challenging enough to engage players. In this paper, we delve into an implementation of the backtracking algorithm in Python for Sudoku puzzle generation. We discuss the algorithm's intricacies, its application in filling Sudoku grids methodically, and how Python facilitates an elegant and efficient solution to this classic puzzle generation problem.

The backtracking algorithm, renowned for its ability to systematically explore solutions while adhering to constraints, proves instrumental in Sudoku puzzle generation. By starting with an empty grid and iteratively attempting to place digits, the algorithm ensures each placement is valid through constraint checks—verifying row, column, and subgrid conditions. When a conflict arises, indicating a dead-end in the current path, the algorithm backtracks to explore alternative choices. This iterative refinement not only guarantees the creation of valid Sudoku puzzles but also allows for varying levels of difficulty by adjusting the number of initial digits (clues) provided. The flexibility and efficiency of implementing backtracking in Python make it an ideal choice for generating Sudoku puzzles that challenge and entertain players of all skill levels.

**EXISTING TECHNIQUES:**

**1. Basic Backtracking with Constraint Propagation**

* **Description:** This combines the basic backtracking approach with constraint propagation, where the algorithm tracks which numbers are valid for each cell. Forward checking is used to eliminate invalid numbers from the potential choices before trying them.
* **Benefit:** Reduces unnecessary guesses by narrowing down valid choices, minimizing backtracking.

**2. MRV (Minimum Remaining Values) Heuristic**

* **Description:** This technique selects the cell with the fewest available valid numbers to fill first. By focusing on the most constrained cells, the search space is reduced.
* **Benefit:** It leads to fewer choices and, consequently, fewer mistakes, which minimizes the need for backtracking.

**3. Randomized Backtracking**

* **Description:** In randomized backtracking, numbers are attempted in a random order rather than a fixed sequence (1-9). This ensures variety in the generated puzzles and avoids generating the same puzzle repeatedly.
* **Benefit:** Adds randomness to puzzle generation, ensuring diverse and unique Sudoku puzzles.

**4. Dancing Links Algorithm (DLX)**

* **Description:** DLX is an efficient implementation of Algorithm X for solving exact cover problems. It converts Sudoku into an exact cover problem and then solves it using an optimized data structure called Dancing Links.
* **Benefit:** Very efficient for generating complete Sudoku grids and ensuring valid solutions with minimal backtracking.

**METHODOLOGY:**

The methodology for efficient Sudoku generation using backtracking involves several key techniques designed to optimize the generation process and ensure puzzle variety and solvability. First, **basic backtracking with constraint propagation** is employed, where the algorithm attempts to place numbers in the grid by recursively testing possible values for each cell. To enhance efficiency, constraint propagation methods such as forward checking are used to eliminate invalid numbers from a cell's potential choices, significantly reducing the search space and avoiding unnecessary backtracking.

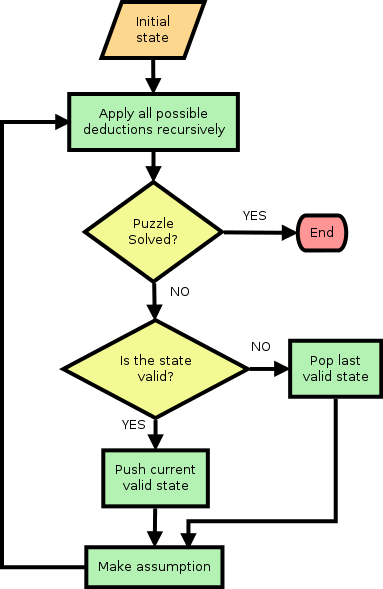
Next, the **Minimum Remaining Values (MRV) heuristic** is applied, which prioritizes cells with the fewest available options, focusing the algorithm's efforts on the most constrained areas of the puzzle. This approach ensures that decisions are made where there are fewer possible choices, leading to fewer conflicts and backtracking steps.

To introduce variety in the generated Sudoku puzzles, **randomized backtracking** is utilized. Instead of trying numbers in a fixed order, the algorithm selects numbers randomly for each cell. This randomness ensures that each puzzle generated is unique and avoids producing the same or similar puzzles repeatedly.

Finally, the **Dancing Links (DLX) algorithm** is implemented to further optimize the process. DLX solves Sudoku puzzles by converting the problem into an exact cover problem, which is then solved using an efficient data structure. This method significantly reduces the number of steps required to generate a valid solution, making it one of the most effective approaches for Sudoku generation.

Together, these techniques streamline the generation process, ensuring that Sudoku puzzles are generated quickly, efficiently, and with a guarantee of solvability.

**FLOW CHART:**



**PROGRAM:**

#include <stdio.h>

#include <stdlib.h>

#include <math.h>

#include <stdbool.h>

#include <time.h>

#define N 9 // Size of the Sudoku grid

#define K 40 // Number of cells to remove

int mat[N][N]; // Sudoku grid

int SRN; // Square root of N

int main() {

SRN = (int)sqrt(N); // Compute square root of N

int i,j,x,y,a,b,num,k;

// Fill the diagonal of SRN x SRN matrices

for ( i = 0; i < N; i += SRN) {

for ( j = 0; j < SRN; j++) {

int num;

for ( x = 0; x < SRN; x++) {

for ( y = 0; y < SRN; y++) {

while (true) {

num = rand() % N + 1; // Generate a random number between 1 and N

bool safe = true;

// Check if num is used in the current box

for (a = 0; a < SRN; a++) {

for (b = 0; b < SRN; b++) {

if (mat[i + a][j + b] == num) {

safe = false;

break;

}

}

}

if (safe) {

mat[i + x][j + y] = num; // Place the number

break; // Exit while loop

}

}

}

}

}

}

// Fill remaining blocks

int filled = 0;

for (i = 0; i < N; i++) {

for ( j = 0; j < N; j++) {

if (mat[i][j] == 0) {

for ( num = 1; num <= N; num++) {

bool safe = true;

// Check if num is safe to place

for ( k = 0; k < N; k++) {

if (mat[i][k] == num || mat[k][j] == num) {

safe = false;

break;

}

}

// Check in the box

for (a = 0; a < SRN; a++) {

for ( b = 0; b < SRN; b++) {

if (mat[i - i % SRN + a][j - j % SRN + b] == num) {

safe = false;

break;

}

}

}

if (safe) {

mat[i][j] = num; // Place the number

filled++;

break; // Move to the next cell

}

}

}

}

}

// Remove K digits to create the Sudoku puzzle

srand(time(NULL)); // Seed the random number generator

int count = K;

while (count != 0) {

int i = rand() % N; // Random row

int j = rand() % N; // Random column

if (mat[i][j] != 0) {

mat[i][j] = 0; // Remove the number

count--;

}

}

// Print the Sudoku grid

for (i = 0; i < N; i++) {

for ( j = 0; j < N; j++) {

printf("%2d ", mat[i][j]);

}

printf("\n");

}

return 0;

}

**Output:**

0 6 0 6 5 1 0 0 0

5 0 0 8 2 0 6 0 0

0 0 0 3 7 0 5 8 0

9 6 6 0 0 7 0 2 5

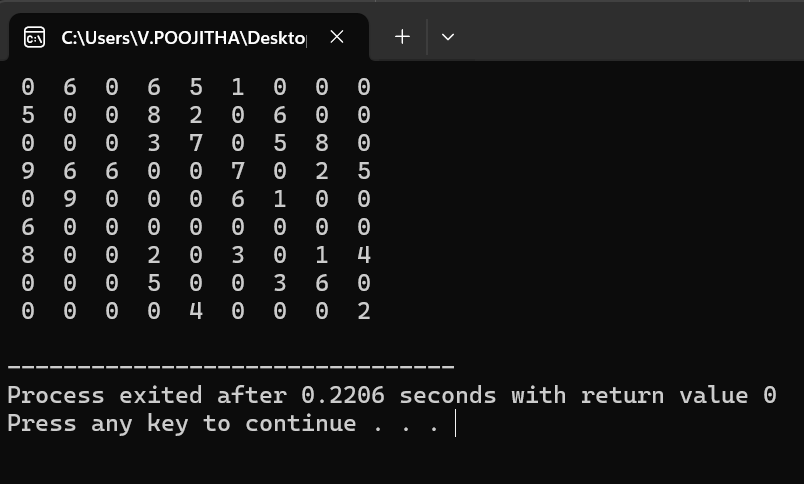
0 9 0 0 0 6 1 0 0

6 0 0 0 0 0 0 0 0

8 0 0 2 0 3 0 1 4

0 0 0 5 0 0 3 6 0

0 0 0 0 4 0 0 0 2



**COMPLEXITY ANALYSIS**

The time complexity of generating a Sudoku puzzle using the backtracking algorithm is primarily influenced by the number of valid placements at each cell. In the worst-case scenario, the algorithm may need to explore every possible combination of numbers, leading to an exponential time complexity of O(N^N^2)O(N^{N^2})O(NN2), where NNN is the grid size (9 for standard Sudoku). However, due to the constraints imposed by Sudoku rules (each number must be unique in its row, column, and 3x3 sub-grid), the average case is significantly better. In practice, the backtracking algorithm typically operates with an average time complexity of O(N^2)O(N^2)O(N^2) to O(N^3)O(N^3)O(N^3) due to the rapid pruning of invalid configurations.

* **BEST CASE:**

The best-case scenario occurs when the algorithm quickly fills the Sudoku grid with minimal backtracking required. This can happen if the algorithm encounters a situation where many cells have straightforward, unambiguous placements. In this case, the time complexity can be close to O(N^2)O(N^2)O(N2), where the algorithm efficiently places numbers without having to explore many alternatives.

* **WORST CASE**

## The worst-case scenario arises when the algorithm must explore numerous possibilities due to many conflicts in placements, leading to extensive backtracking. This could occur in configurations where valid numbers are scarce for certain cells, forcing the algorithm to backtrack repeatedly. Consequently, the time complexity in this case can approach O(N^N^2)O(N^{N^2})O(N^N^2), as the algorithm may need to evaluate every possible arrangement before finding a solution.

## **AVERAGE CASE**

On average, the performance of the backtracking algorithm tends to be more favorable than the worst case due to the pruning of invalid placements. While the precise time complexity can vary based on the specific board configuration, it generally falls within the range of O(N^2)O(N^2)O(N2) to O(N^3)O(N^3)O(N^3). This is because the algorithm efficiently eliminates many invalid options, allowing it to fill the Sudoku grid in a reasonable amount of time for most randomly generated puzzles.

**FUTURE SCOPE**

### The future scope for efficient Sudoku generation using backtracking includes exploring advanced algorithms, such as Constraint Satisfaction Problems (CSP) solvers and heuristic-based methods, to improve puzzle generation speed and variety. Additionally, integrating artificial intelligence could enhance puzzle difficulty adjustment based on user preferences, while dynamic graphical user interfaces could provide interactive solving experiences. Expanding into Sudoku variants, optimizing performance with parallel processing, and developing educational tools to teach logic and problem-solving skills are also promising directions. These advancements can create more engaging, diverse, and user-friendly Sudoku experiences, appealing to a broader audience.

### **CONCLUSION**

In conclusion, Efficient Sudoku generation using backtracking presents a valuable approach to creating engaging and challenging puzzles while ensuring a unique solution. By leveraging the power of backtracking, the algorithm not only generates complete Sudoku boards but also allows for the random removal of digits to create playable puzzles. Despite variations in performance across best, worst, and average cases, the overall adaptability and effectiveness of the backtracking method make it a robust choice for Sudoku generation. With opportunities for further enhancements and innovations, the future of Sudoku generation holds exciting potential for both recreational and educational applications.

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2. **Knuth, D. E. (2000). "Dancing Links." In *The Art of Computer Programming* (Vol. 4, Fascicle 5, pp. 8–9).**Donald Knuth introduces the Dancing Links (DLX) algorithm as an efficient way to solve exact cover problems, which can be applied to Sudoku puzzle generation. This is a foundational reference for understanding how the DLX algorithm works in Sudoku generation.
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